MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

May 2010

Designing the Mathematics Standards for Years 1-8

Why were the standards created?

In December 2008 legislation was passed giving the Minister of Education the power to set national standards in reading, writing and mathematics. The standards were to set clear expectations for progress and achievement in reading, writing and mathematics for students in years 1-8.

Background

The standards for mathematics were developed alongside the development of the standards for reading and writing. Clearly, as much as possible, both sets of standards had to meet a common design specification. The approach was guided by a set of principles agreed to by a group of experts in mathematics education, literacy and assessment convened by the Ministry in December 2008 (see Appendix 1).

What was the rationale for the form of the standards?

A mathematics standards "critical friends" group convened in 2009. It comprised key people in the mathematics education and assessment communities, and established the following guidelines and rationale for the creation of the standards.

1. Standards are broad statements of valued outcomes supported by ways to measure those outcomes.

The New Zealand Curriculum (Ministry of Education, 2007) and supporting resources provide teachers with adequate detail in developing fine grained learning outcomes for students[1]. Overseas experience indicated that standards based on large numbers of specific outcomes were associated with excessive assessment and fragmentation of mathematical content.

2. Standards reflect the philosophy and content of the mathematics and statistics learning area of *The New Zealand Curriculum*.

The development of the mathematics and statistics area involved three years of consultation and development. In meeting the philosophy of the curriculum the standards reflect the emphasis on key competencies, on modelling and problem solving in context across all strands and the progressions described through the levels.

3. Standards define achievement in mathematics by students' ability to solve problems and model situations in context.

In the standards, knowledge is deemed to be in the service of problem solving and modelling in keeping with the approach of the reading and writing standards, and of modern mathematics curricula worldwide. This approach is not intended to demean the importance of students learning key knowledge but emphasises the pointlessness of knowing without understanding and without the ability to apply that knowledge. In this sense the possession of knowledge without the ability to apply it does not meet expectations for the standards.

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[1] For example, Numeracy Development Projects materials, and mathematics curriculum support material online at www.nzmaths.co.nz

4. Standards describe both levels of achievement and rates of progress.

The levels of *The New Zealand Curriculum* were set through reference to considerable norm-based research evidence about appropriate difficulty of concept and problem type. The achievement objectives and levels of *The New Zealand Curriculum* describe progression that is supported by research-based developmental frameworks.

5. Standards are set at a level of difficulty that reflects what can reasonably be achieved by students given quality instruction.

The intention of standards is to raise student achievement. This can only occur if standards reflect high yet reasonable levels of achievement. Data from the longitudinal study of long-term Numeracy Development Project schools (Thomas & Tagg, 2007) suggested that student achievement, particularly in the later years of primary school, was higher in schools where the project philosophy and approaches became a fundamental part of school culture than was the national norm. A continued focus on using data to improve student outcomes was a key feature of these schools.

Other factors impacted on the design of the mathematics standards. For consistency the mathematics standards accepted the schooling points set for the literacy standards[2]. These points were based on existing assessment practice in New Zealand primary schools.

The policy requirement for standards at each year level required a differentiation of each curriculum level into two sets of standards. For example, the achievement objectives of level three were differentiated into the standards for years five and six. This differentiation was done though reference to norm-referenced item information from standardised tests and tasks, to research-based developmental frameworks, and to predicted bridging between achievement objectives from consecutive levels. This bridging involved use of known variables in task difficulty such as the use of inverse operations, number size and referent complexity, requirements for multiple steps and the simultaneous connection of spatial features.

In the last decade the Ministry of Education has invested heavily in the professional development of teachers through the Numeracy Development Projects, in resources for teachers and students such as *Figure It Out*, and in assessment tools like the diagnostic interview from the Numeracy Development Projects, and through initiatives such as the *National Education Monitoring Project* (NEMP), *Assessment Tools for Teaching and Learning* (AsTTle), the *Assessment Resource Banks* (ARBs) and the *Assessment Exemplars*. The approaches and resources from these initiatives form a common infra-structure for mathematics and statistics programmes in New Zealand. The standards make full use of this infra-structure in the choice of examples that illustrate expectations.

What was the relationship between the standards and *The New Zealand Curriculum*?

The draft standards prepared for consultation organised the content of the mathematics and statistics learning area around key processes such as quantifying, measuring and classifying. This was in keeping with the aim of creating a small number of global standards.

The majority of feedback from the consultation rejected this approach and called for alignment of the standards with the strands and levels of *The New Zealand Curriculum*. Many respondents also expressed the need for synergy between the structure of common assessment tools, mostly based on strands and levels, and the standards.

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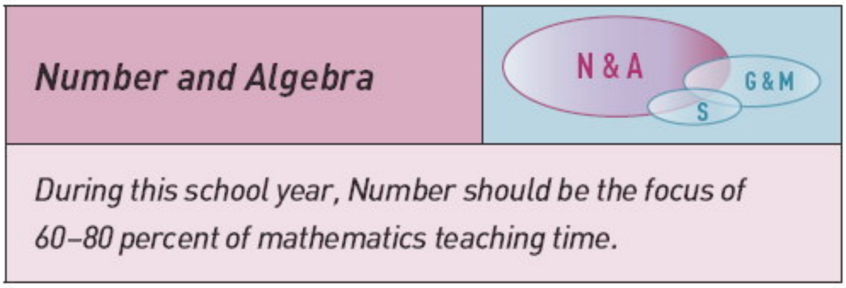
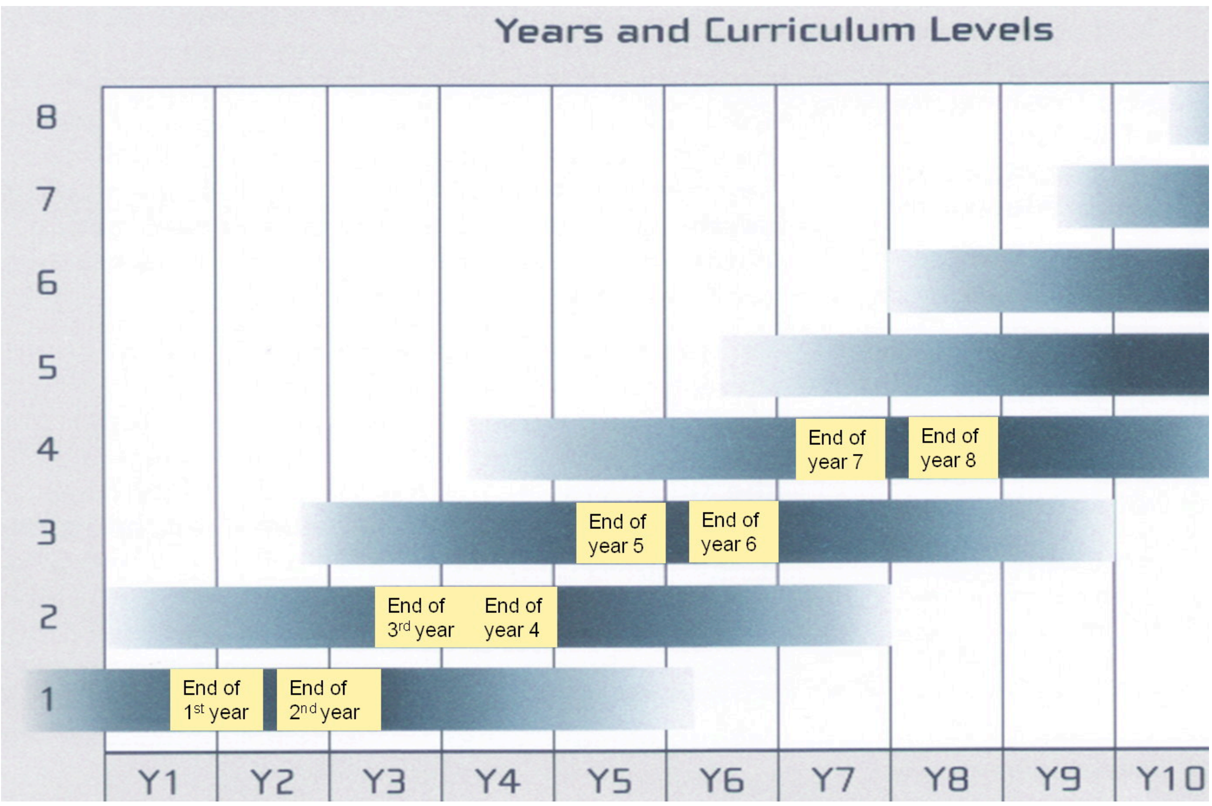
[2] See page 4 of *Designing the Reading and Writing Standards for Years 1-8*

In keeping with this feedback the standards are organised by the strands of *The New Zealand Curriculum*; number and algebra, measurement and geometry and statistics. Venn diagrams from *The New Zealand Curriculum* are used to describe the relative teaching, and therefore assessment emphasis, that should be placed on the three strands at different schooling points. The diagram below illustrates the balance expected for the end of the third year of schooling.



The learning progressions described by the levels of *The New Zealand Curriculum* provide the core structure for the standards at various schooling points. The relationship between the levels and expectations for each schooling point is shown overleaf. The levels diagram means that at any given schooling point students will be at a range of levels. The heavily shaded area of each level band describes the year or years at which the majority of students are expected to achieve the outcomes of that level. An obvious implication of this is that the standard for any schooling point will be too difficult for some students, appropriate for the majority of students and too easy for other students at the matching age or class level. However, the progressive nature of the standards allows teachers, parents and students to choose standards that are appropriate to the individual student and to provide ambitious learning goals.



How were the standards set?

The number and algebra standards were set with reference to several sources:

1. Student achievement data from the Numeracy Development Projects since 2000 provided a base for defining high but reasonable expectations. The data from the long term Numeracy Development Project (longitudinal schools) showed higher levels of achievement, particularly in the later years of primary school, than the data from schools after one year on the project. A logical implication is that the approaches of the project take teachers several years to fully implement. This assumption aligns with the results of the Best Evidence Synthesis (Timperley, Wilson, Barrar & Fung, 2007). The standards expectations were set at a level that was currently achieved by approximately two-thirds of students in classes taught by teachers in the first year of the Numeracy Development Project. The exception to this benchmark is at years 7 and 8 where the expectations are currently met by just over one half of students.
2. National standards overseas provided a set of comparative expectations. Particular attention was paid to the standards developed in countries or states where marked gains in student achievement had been recorded over the last five years, (Department of Education, Minnesota, 2007, Ministry of Education, Ontario, 2005). Successful countries and states appeared to align standards that reflected very high levels of expectation with the professional development programmes offered for teachers, assessment practices used and additional support for students in need.
3. The National Education Monitoring Project (NEMP), Assessment Tools for Teaching and Learning (AsTTle), and the Progressive Achievement Tests (PAT) for Mathematics provided rich data about student success on assessment tasks.

Given the necessity to differentiate within each level of the curriculum, and by implication each stage of the Number Framework, task variables were used to define increased difficulty. Confidence in this process was gained through access to case study research on student number acquisition and from the relative difficulty of the interview tasks from the Numeracy Development Project assessment tools. Known task variables are number size[3], complexity of the operation (e.g. multi-step, use of inverse operations, additive or multiplicative) and the degree of abstraction required (Ellemore-Collins & Wright, 2009).

An example of the use of task variables is given below where the expectations in number and algebra are compared for the end of years 5 and 6. These expectations describe increased sophistication at Level 3 of *The New Zealand Curriculum* and within Stage 6 (Advanced Additive/Early Multiplicative) of the New Zealand Number Framework (Ministry of Education, 2007a). The major points of difference between years 5 and 6 are the use of inverse operations, a command of the properties of subtraction as well as addition, and the solving of multi-step problems involving addition and subtraction and simple multiplication and division. The expectation for algebra is that these strategies will be

[3] Largeness of numbers is not the defining variable. Fractional numbers are known in most cases to be more difficult for students than whole numbers. It is the interaction between the place value structure of the numbers and the operations required that creates variation in task difficulty.

transferred to predicting further members of relations (ordered pairs) from patterns and describing rules for the observed relationships using multiple representations (e.g. graphs, tables).

By the end of year 5, students will be achieving at early level 3 in the mathematics and statistics learning area of the New Zealand Curriculum.

Number and Algebra

N & A G&M S

In contexts that require them to solve problems or model situations, students will be able to:

- *apply additive and simple multiplicative strategies and knowledge of symmetry to:*
  - *combine or partition whole numbers*
  - *find fractions of sets, shapes, and quantities;*
- *create, continue, and predict further members of sequential patterns with two variables;*
- *describe spatial and number patterns, using rules that involve spatial features, repeated addition or subtraction, and simple multiplication.*

By the end of year 6, students will be achieving at level 3 in the mathematics and statistics learning area of the New Zealand Curriculum.

Number and Algebra

N & A G&M S

In contexts that require them to solve problems or model situations, students will be able to:

- *apply additive and simple multiplicative strategies flexibly to:*
  - *combine or partition whole numbers, including performing mixed operations and using addition and subtraction as inverse operations*
  - *find fractions of sets, shapes, and quantities;*
- *determine members of sequential patterns, given their ordinal positions;*
- *describe spatial and number patterns, using:*
  - *tables and graphs*
  - *rules that involve spatial features, repeated addition or subtraction, and simple multiplication.*

The measurement and geometry expectations are based on a less consistent body of literature. In measurement, progression is determined by increased sophistication in the use of a unit of measure (Lehrer, Jaslow & Curtis, 2003) and the perceptual difficulty of the attribute being measured[4]. Lehrer et al's work strongly influenced the measurement progression in both *The New Zealand Curriculum* and the standards. Stages of development are: direct comparison; use of a unit of measure (informal to formal); development of scale; reasoning with measures; and relationships between measures. This framework was correlated with assessment data from NEMP, AsTTle, PAT and the Assessment Exemplars to establish suitable expectations. Consideration was also given to computational competency assumed in the number and algebra expectations and the likely impact of this competency on solving measurement problems[5].

Similar processes were used for creating the geometry expectations. Research based frameworks (e.g. Van Hiele & van-Hiele-Geldoff, 1984) define progression in spatial reasoning by increasingly sophisticated classification systems. Classification increases in sophistication from non-attendance to properties, to visual attendance to global similarity, to attendance to properties, to establishment of classes and sub-classes based on those properties, to reasoning from the properties alone (Ministry of Education, 2007e).

Little is known about task variables that influence the complexity of spatial visualization tasks such as representing three-dimensional objects using two-dimensional diagrams. The standards drew heavily on the data from assessment tools to establish the attributes of tasks that were successfully answered

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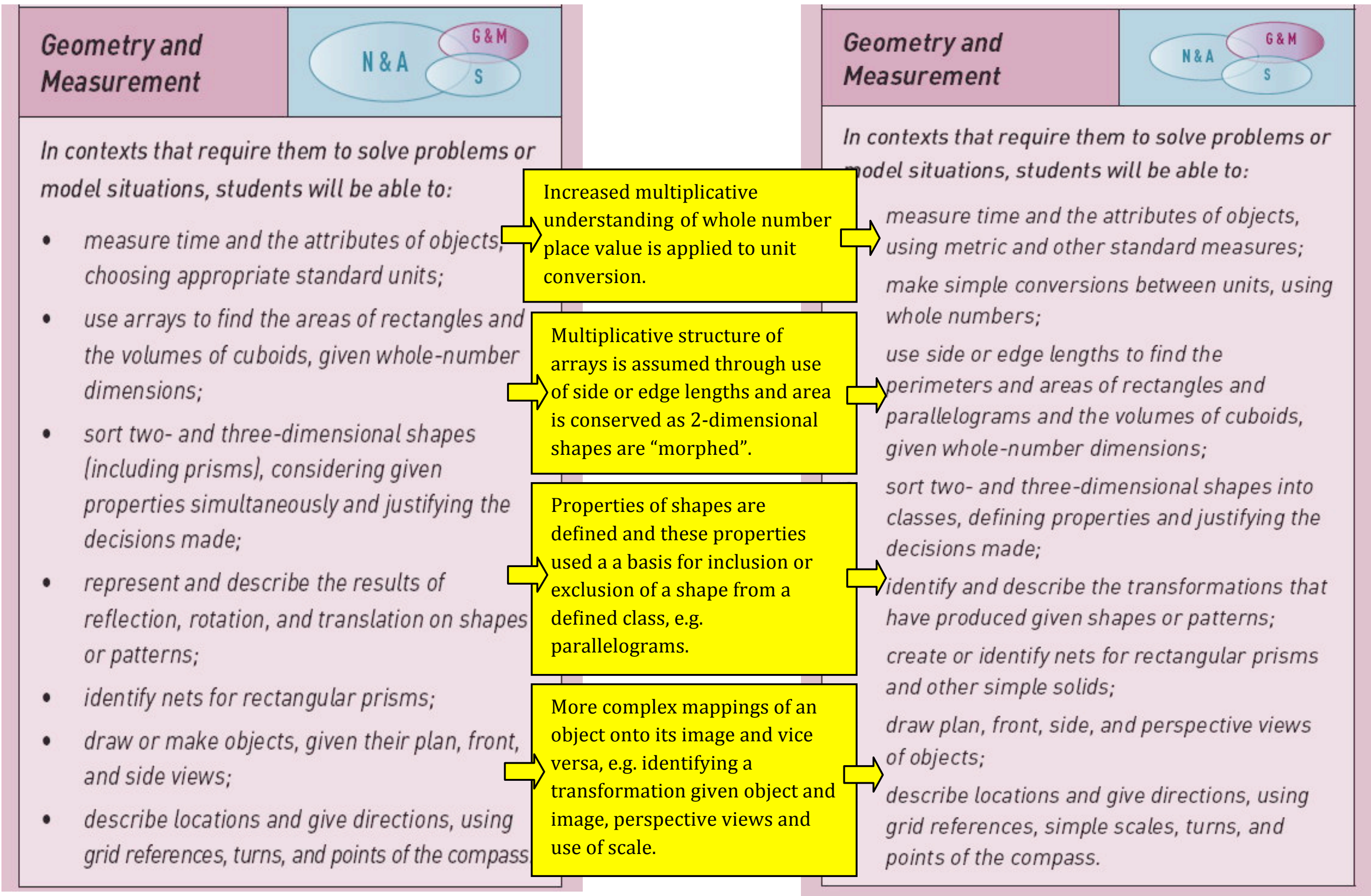
[4] Research since Piaget's early work has highlighted that length is perceptually easier than area and volume. Conservation of "non-tactile" attributes such as weight and time tends to occur later in student development than conservation of attributes that can be seen and felt.

[5] Examples of this relationship include the finding of areas and volumes using counting, additive and multiplicative computational strategies and the impact of decimal knowledge on conversions between units of measure.

by about two-thirds of students at each schooling point. This analysis of task difficulty did suggest that the complexity of mapping between an object and its image was a potentially fruitful avenue of study for future research.

The figure below shows how the principles used in describing progression between the standards for consecutive years were applied to geometry and measurement at years 6 and 7.



The expectations for statistics reflect the progression developed for *The New Zealand Curriculum*. Throughout the standards the critical place of the statistical inquiry cycle is reiterated. The defining characteristics for progression from one standard to the next are the nature of the data being used[6], the nature of the analysis being applied to the data[7], and the sophistication of relating the results to the original context.

The progression through the standards in probability reflects case study research into learner progression (Jones et al, 1997, Shaughnessy, 1992). Student responses to NEMP and assessment

[6] For example, category data is referred to at the end of year one, simple whole-number data is referred to for the end of year three.

[7] For example, the expectation at the end of year 5 is to “gather, display, and identify patterns in category and whole-number data” and for the end of year 6 is to “sort data into categories or intervals, display it in different ways, and identify patterns”. This progression involves understanding the significance of category and interval choice in detecting patterns.

exemplar items were mapped to the progression framework to establish appropriate expectations for each schooling point.

For both statistical investigation and probability the standards exercise revealed the limitations of closed and multi-choice response items in assessing student understanding[8]. Statistical literacy expectations are not specifically stated in the standards. This aspect of *The New Zealand Curriculum* is assumed to be encompassed in the exercise of statistical investigation and should be a key aspect of the literacy across the curriculum.

Where were the illustrations of the standards sourced from?

The standards draw on the following learning and assessment resources that are key components of the national infra-structure:

- The Figure It Out series
- Progress and Achievement Tests (PAT)
- Numeracy Project Assessment Tools (NumPA, GloSS)
- Assessment Exemplars
- National Education Monitoring Project (NEMP)
- Mathematics Curriculum Support Material (http://www2.nzmaths.co.nz/frames/curriculum/index.aspx)

The choice of examples to illustrate the expectations reflects the goal of the Critical Friends group that the standards reflect the best of current practice in New Zealand schools.

[8] Statistical investigation is a dynamic process that requires students to work through a prolonged data-based investigation using the inquiry cycle. Limited response items can assess restricted examples of the competencies required, e.g. asking questions, choosing appropriate displays, making statements. Probability requires a dynamic interaction between theoretical and experimental models.

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Appendix 1

Principles agreed to by consultation group 18/12/08

Principles (Related to identifying and measuring standards)
• Measure what is important in a way that enables effective instruction for growth and development for all students. • Standards are based on a psychometrically defensible scale, multiple measures and the best available evidence. • Communicability, accessibility, utility, transparency for a range of audiences will be key considerations in the development of standards. • Standards will require a well-informed mix of actual and aspirational tasks. • Standards will contribute to system improvement. • Standards will enable understanding of individual trajectories of development to promote progress and growth. • The least well-served students will be better off in relation to valued outcomes of *The New Zealand Curriculum.* • Integral part of curriculum teaching and learning. • Standards trigger a response from all levels of the system for learners. • Decisions about where student is at in relation to standards are drawn from multiple sources. • Standards will be supported by exemplification in the form of multiple authentic student work showing what achievement could look like.